A Unification Scheme for Classical and Quantum Mechanics at All Velocities

— The Fundamental Formation of Material Particles & Consistent Theory of Relative

Motion

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Abstract

We review in Part I the whereabouts, especially the unsolved problems from the point of view of a united theoretical basis, in the issues of physics relevant to here, these including the interpretation of Schrödinger's wave function, the interpretation of Heisenberg's uncertainty relation, the nature of inertial mass, space and time, the nature of elementary particles with an overview of the pictures given in quantum field theory and in string theory, and the existing unification efforts (chiefly made towards unification of the four forces), etc. This lays the background from which we embarked our unification project since mid 2000.

Parts II-III deal with the main theme of this monograph, where we introduce our recently achieved result, *The Unification of Classical and Quantum Mechanics*. Our approach is to firstly derive a realistic submicroscopic model for vacuum (of a Dirac kind) based on overall experimental observations, establish then the Newtonian equation of motion of the vacuum under an external perturbation, and solve.

Part II introduces the first part of the unification scheme, **The fundamental formation of martial particles**. A systematic survey of experimental indications leads us to conclude that vacuum consists of electrically neutral, strongly bound bare-charge pairs, each pair consisting of a positive bare-charge and a negative bare-charge. Our Newtonian solution for the so structured vacuum shows that, a basic particle, which may be e.g. an electron, is composed of a tiny free *bare-charge* and the mechanical wave disturbances – identifying with electromagnetic waves – generated by it in the (vacuum) medium. When in motion, at a velocity v (here $(v/c) \to 0$), as a result of a *first kind source effect* this particle wave exhibits all of wave and dynamic properties known for a de Broglie wave, and is here called a *Newton- de Broglie* (NdB) particle wave. In a confined space, the Newtonian solution for the NdB particle wave is equivalent to that given by Schrödinger's quantum mechanics. Through this *general scheme* for particle formation we have accomplished a basic task of the *unification of the classical- and the quantum- mechanics*, both in terms of the deduction of the latter from the former, and the convergence of the latter into the former at high velocities. And we unfold the origins and nature of a series of phenomena including the *electromagnetic waves*, the electromagnetic *radiation* and *absorption*, *atomic* and *thermal excitations*, the *inertial mass*, the *Schrödinger's wavefunction* and *de Broglie wave*, the *Heisenberg's uncertainty relation*, the *de*

Broglie relations, the *simultaneous existence of electron and positron* or generally of *particles and their anti-particles*, the *(rest) mass-energy equivalence* relation, etc. The most recently achieved result, a derivation of Schrödinger's equations from Newtonian mechanics will be presented in one chapter.

Part III presents the second part of the unification scheme, **The Theory of Relative motion**. We show in Chapter 6 that the $(v/c)^2$-dependent terms yield to the particle wave and dynamic quantities a *second kind source motion effect* (SSME). The SSME augments the particle mass, and the wavevector and frequency of the particle's constituent waves, etc., by a factor $gamma = 1/\sqrt{1 - v^2/c^2}$ in the v-direction; and conversely for the reciprocal quantities. Subsequently a moving body comprising the so affected particles will present a *simultaneous length and time contractions* (of Lorentz-Fitzgerald kind) owing purely to the SSME. A systematic survey of the relevant pivotal experimental indications in Chapter 7 leads us to conclude that the light velocities, c measured in vacuum and c' measured by a moving observer, and the observer's velocity v obey the common *triangle law of vector addition*, conforming to the Galilean transformation (GT). Combining the conclusions of Chapters 6-7 yields a set of transformation equations between an inertial reference frame at rest (in vacuum) and one moving relative to it, called *Galilean-Lorentz transformation* (GLT). The GLT together with the underlying theoretical basis of the *general scheme* yields a consistent *Theory of Relative Motion*. With the theory, we predict the observational null-/constant- fringe shift result of the Michelson-Morley/Kennedy-Thorndike experiment, the Doppler effects of electromagnetic waves, the equivalence principle of Newton's laws of motion in all inertial frames, etc, and we extend the classical and quantum mechanics to $(v/c)^2 >> 0$. The complete agreement is in turn a justification of the *general scheme*.

Part IV introduces our most recently achieved result within the framework of the *general scheme*, the formulation of a Microscopic Theory of Gravity and the derivation of Newton's Gravitational Law. We also discuss the effective means for detecting the gravitational wave.

In the end we outline the planned future research problems within our unification projects, these including the explanation of the missing anti-protons, the derivation of Pauli exclusion principle, the development of a theory of relative motion in non-inertial frames, and a treaty of cosmological problems.

Contents

Part V

Microscopic Theory of Gravity

Chapter 11

The Microscopic Theory of Gravity

11.1 The origin of gravity

11.1.1 The two key properties of matter-vacuum leading to gravity

We now elucidate that gravity originates from the two key properties of the matter–vacuum system as recognized through the *general scheme* for particle formation.

Firstly, by the *general scheme*, a basic material particle is composed of an oscillatory vaculeon (a signed elementary charge) and the (virtually radiated) electromagnetic waves generated by the vaculeon. So two basic material particles, 1 at $\mathbf{r}_1$, and 2 at $\mathbf{r}_2$, will always interact with each other via their constituent electromagnetic waves, of electric field $\mathbf{E}_{qi}$ and $\mathbf{B}_{qi}$, $i = 1, 2$; this is in addition to the electrostatic interaction between the two vaculeons charges. In respect of the origin of gravity, only the former interaction is relevant and thus be the subject of discussion for this chapter. These fields were shown in the preceding chapters to be directly coupled to the particles' mass, which gives the first signature of its connection with gravity.

Secondly, as also stated in the general scheme, the vacuum is a *dielectric* medium. With reppec to this dielctric medium there firstly ought be a In this dielectric medium the radiated field $\mathbf{E}_{qi}$ will induce **"empty " space** — the space after subtracting the dielectric vacuum, in which the radiated field will be $\mathbf{E}_q^*$. $\mathbf{E}_q^*$ will secondly induce in the dielectric medium a **depolarization field**, $\mathbf{E}_{\rm pol}^*$, which is always opposite to the direction of $\mathbf{E}_q^*$. Their relation, as well as other overall properties of the dielectric vacuum, is formally represented in Appendix B. The two opposite electric fields produced by particle 1 in vacuum, when combined with its magnetic field $\mathbf{B}_{q1}$, will act on particle 2 two opposite Lorentz forces, which we inspect below. The impression of a force of one particle's radiated fields on another particle, is the second signature of its connection with gravitational force.

11.1.2 Lorentz force due to external-charge field: The radiation force

Concerning the first component field, in classical electromagnetic theory for a "empty vacuum", it is well established that the thermally radiated fields $\mathbf{E}_{q.1}$ (or more actually by $\mathbf{E}^*_{q1}$) and $\mathbf{B}_{q1}$, of particle 1 will always act on particle 2 a Lorentz magnetic force, $\mathbf{F}_m$; which is known as the radiation pressure on an macroscopic scale (i.e. when concerning the interaction of many particles). $\mathbf{F}_m$ is as determined by the right-hand rule always a repulsive force, $\mathbf{F}_m$; see Figure F, Appendix F. In other words, it is always in the direction of the wave propagation, or the direction of the Poynting vector (the vector intensity of electromagnetic energy flux, $\mathbf{I} = \mathbf{E} \times \frac{\mathbf{B}}{\mu_o}$).

11.1.3 Lorentz force due to vacuum depolarization field: The gravitational force

Th depolarization field of vacuum $\mathbf{E}^*_{\text{pol.1}}$ is as a result that vacuum is a dielectric; it would not exist if asserting an "empty" vacuum. $\mathbf{E}^*_{\text{pol.1}}$ is always opposed to the

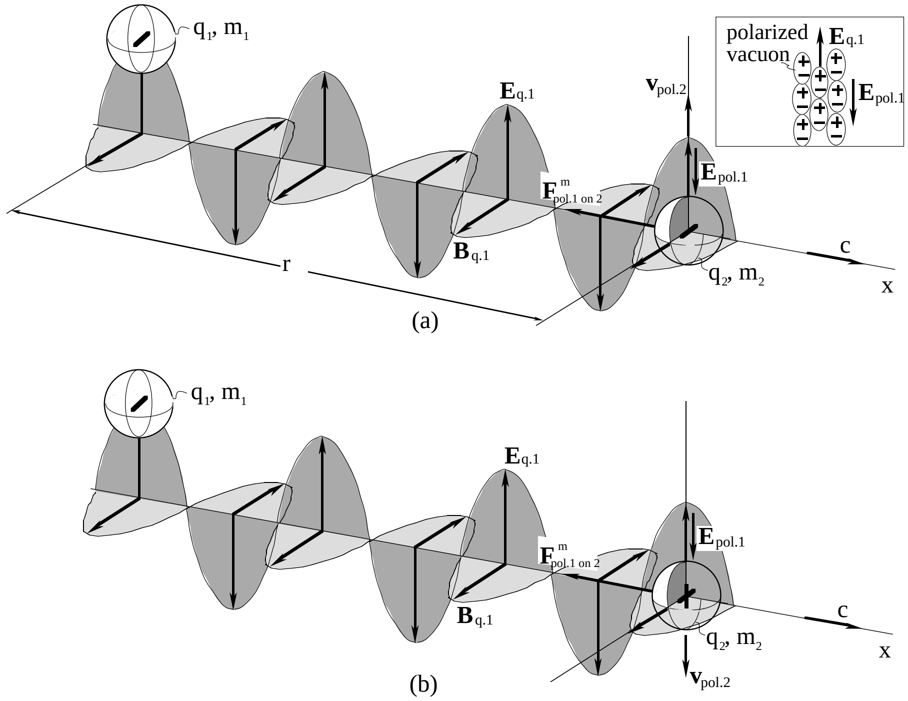

Figure 11.1:

direction of $\mathbf{E}_{q1}$. By Newton's second law combined with Coulomb's law, $\mathbf{E}^*_{\text{pol.1}}$ drives mass 2 into a velocity $\mathbf{v}_{\text{pol.2}}(\mathbf{r}, t)$ in the direction of $\mathbf{E}^*_{\text{pol.1}}$ for positive change (Figure 11.1b), and in reversed diction if negative charge(Figure 11.1b). By Lorentz force law partcile 1 acts on partcile 2 a magnetic force

$$\mathbf{F}_{m.\text{pol.1 on 2}} = q_2 \mathbf{v}_{\text{pol.2}} \times \mathbf{B}_{q1}$$

which may be called a **radiated field induced vacuum depolarization resultant Lorentz force** (RFVP Lorentz force). The direction of $\mathbf{F}_{m.\text{pol.1 on 2}}$ is as determined by the right-hand rule always directed to particle 1. In other words, $\mathbf{F}_{m.\text{pol.1 on 2}}$ is always attractive, and thus always opposite to the direction of radiation force. Figure 11.1 illustrates the direction of this attraction force for two cases: (a) two masses both negatively charged; (b) one negatively charged mass and one positively charged mass. One can check any other system of charged masses, and move the charges to various positions relative to the radiated field function, one will find that an attractive force is always true. As judged by its attractive nature, and the exact prediction of it of the Newtonian law of gravitation in the next section, we can identify the RFVP Lorentz force, $\mathbf{F}_{m.\text{pol.1 on 2}}$, as the universal gravitational force.

11.2 Inference of Newton's Law of Gravitation

Based on the origin of gravity characterized in Sec. 11.1, we below derive the exact expression for the attractive RFVP Lorentz force based on Newtonian mechanics and Maxwellian electromagnetism, and thereby infer Newton's Law of Gravitation.

11.2.1 Gravitational force between two basic material particles

We consider as shown in Figure 11.1a two basic material particles at $\mathbf{r}_1 = 0$ and $\mathbf{r}_2 = \mathbf{r}$ in the vacuum, of charges q_1 and q_2, respectively. Their separation distance is

$$r = |\mathbf{r}_2 - \mathbf{r}_1| = |\mathbf{r}_2|. \tag{11.1}$$

We regard for explicitness the first charge as the source and the second as the test charge; this order can be entirely reversed. By the general scheme for particle formation, each of the the particles, 1 and 2, comprises a vaculeon (elementary charge $q_1 = \pm e$) and the virtually radiated electromagnetic waves generated by it. The virtually radiated electromagnetic waves of particle q_1, of fields $\mathbf{E}_{q.1}$ and $\mathbf{B}_{q.1} = \mathbf{E}_{q.1}/c$ propagated to $\mathbf{r}_2$ are given following (3.73)–(3.75) to be:

$$\mathbf{E}_{q.1}(\mathrm{r}_2, \theta_2, t) = \frac{\mu_0 q_1 \omega_1^2}{4\pi} \frac{A_1 \sin\theta_2}{r} \sin(\omega_1 t - k_1 r)\hat{\theta}_2 \tag{11.2}$$

$$\mathbf{B}_{q.1}(\mathrm{r}_2, \theta_2, t) = \frac{\mathbf{E}_{q.1}(\mathrm{r}_2, \theta_2, t)}{c} = \frac{\mu_0 q_1 \omega_1^2}{4\pi c} \frac{A_1 \sin\theta_2}{r} \sin(\omega_1 t - k_1 r)\hat{\phi}_2 \tag{11.3}$$

Where μ_0 is the permeability of vacuum, c light velocity in vacuum, ω_1 and k_1 the angular frequency and wavevector of particle-1's constituent electromagnetic wave and A_1 the displacement amplitude of the apparent vacuon chain at the vicinity of the vaculeon of particle 1.

$\mathbf{E}_{q.1}$ induces at $\mathbf{r}_2$ a depolarization field of the dielectric vacuum, given by (B.25) as:

$$\mathbf{E}^*_{\text{pol}.1}(\mathbf{r}_2, t) = -\vartheta \mathcal{X}^*_{\text{v}} \mathbf{E}_{q.1}(\mathbf{r}_2, t). \tag{11.4}$$

where $\vartheta = \mathbf{E}^*_{q1}/\mathbf{E}_{q.1}$ as defined in (B.19)$'$, and $\mathcal{X}^*_{\text{v}}$ is the susceptibility of vacuum defined in (B.14), Appendix B. $\mathbf{E}^*_{\text{pol}.1}$ is always opposed to the direction of $\mathbf{E}_{q.1}$, reflected by the minus sign in (11.4). $\mathbf{E}^*_{\text{pol}.1}$ in turn acts an electrostatic force $\mathbf{F}_{e.\text{pol}.1 \text{ on } 2} = q_2\mathbf{E}^*_{\text{pol}.1}$ on chareg q_2, and driving it to a velocity $\mathbf{v}_{\text{pol}.2}(t)$; by Newton's second law:

$$q_2\mathbf{E}^*_{\text{pol}.1}(\mathbf{r}_2, t) = m_2 \frac{\partial \mathbf{v}_{\text{pol}.2}(\mathbf{r}_2, t)}{\partial t} \tag{11.5}$$

The accumulative force in time interval Δt, or momentum, is

$$\Delta t q_2 \mathbf{E}^*_{\text{pol}.1} = m_2 \int_0^{\Delta t} d\mathbf{v}_{\text{pol}.2} = m_2 \mathbf{v}_{\text{pol}.2}(\mathbf{r}_2, \Delta t)$$

$$\text{or} \qquad \mathbf{v}_{\text{pol}.2}(\mathbf{r}_2, \Delta t) = \frac{\Delta t q_2 \mathbf{E}^*_{\text{pol}.1}}{m_2} = -\frac{\Delta t q_2 \vartheta \mathcal{X}^*_{\text{v}} \mathbf{E}_{q.1}}{m_2} \tag{11.6}$$

where the last equation used (11.4).

Menwhile, by Lorentz force law $\mathbf{B}_{q.1}$ acts on the moving charge q_2 a magnetic force:

$$\mathbf{F}_{m.\text{pol}.1 \text{ on } 2}(\mathbf{r}_2) = q_2 \mathbf{v}_{\text{pol}.2} \times \mathbf{B}_{q.1}$$

Substituting (11.6) and (11.3) into it we have:

$$\begin{aligned} \mathbf{F}_{m.\text{pol}.1 \text{ on } 2}(r_2, \theta_2, t) &= \frac{(\Delta t) q_2^2 \vartheta \mathcal{X}^*_{\text{v}} E^2_{q.1}(r_2, \theta_2, \tau)}{m_2 c}(-\hat{r}) \\ &= \frac{(\Delta t)\vartheta \mathcal{X}^*_{\text{v}} \mu_0^2 q_1^2 q_2^2 \omega_1^4 A_1^2}{(4\pi)^2 m_2 c} \frac{\sin^2\theta_2}{r^2} \sin^2(\omega_1 t - k_1 r)(-\hat{r}) \end{aligned} \tag{11.7}$$

We are interested in the average force in a time interval long compared to $1/\omega_1$; also we consider mass 2 is sufficiently large so that effectively all radiated component electromagnetic waves of mass 1 impinging on mass 2. Thus the total average force is given by the time integration over say a wave period τ, distance integration over say a wavelength λ, and θ_2 integration over the entire angle range $0, \pi$. This average of (11.7) writes

$$\mathbf{F}_{m.\text{pol}.1 \text{ on } 2}(r_2) = \frac{1}{\lambda}\frac{1}{\tau} \int_0^{\theta_2} d\theta_2 \int_0^{\lambda} dx \int_0^{\tau} \mathbf{F}_{m.\text{pol}.1 \text{ on } 2}(r_2, \theta_2, t)\ dt \tag{a}$$

The related integration has the result (cf. (M.11), (M.10)):

$$\int_0^{\theta_2} d\theta_2 \int_0^{\lambda} dx \int_0^{\tau} \sin^2\theta_2 \sin^2(\omega_1 t - k_1 r)\delta t = \frac{\pi}{4}$$

With this in (a) gives

$$\mathbf{F}_{m.\text{pol.1 on }2}(r_2) = \frac{\pi}{4}\frac{(\Delta t)\vartheta\mathcal{X}^*_{\text{v}}\mu_0^2 q_1^2 q_2^2 \omega_1^4 A_1^2}{(4\pi)^2 m_2 c}\frac{1}{r^2}(-\hat{r}) \qquad (11.7)'$$

Further, applying (the relativistic form of) (4.26a) to particle 1 gives:

$$\omega_1^2 A_1^2 = \frac{m_1 c^2}{N_{b,L_\varphi}\frac{1}{2}b\rho} = \frac{m_1 c^2}{L_\varphi \frac{1}{2}\rho} = \frac{m_1 c}{\Delta t \frac{1}{2}\rho} \qquad (11.8)$$

Where N_{b,L_φ} is the number of effective vacuon oscillators in the apparent vacuon chain of length L_φ; L_φ equals the distance the wave propagates in time Δt; thus $L_\varphi = c\Delta t$. For the basic material particle here:

$$q_1, = \pm e; \qquad q_2 = \pm e; \qquad q_1^2 = q_2^2 = e^2 \qquad (11.9)$$

Substituting (11.8) and (11.9) into (11.7)′:

$$\mathbf{F}_{m.\text{pol.1 on }2}(r_2) = \frac{\pi}{4}\frac{(\Delta t)\vartheta\mathcal{X}^*_{\text{v}}\mu_0^2 e^4 \omega_1^2 (m_1 c)}{(4\pi)^2 m_2 c(\Delta t\frac{1}{2}\rho)}\frac{1}{r^2}(-\hat{r})$$

Sorting, we have the final formal of the RFVP Lorentz force of mass 1 acting on mass 2:

$$\mathbf{F}_{m.\text{pol.1 on }2}(r_2) = \frac{\pi}{4}\frac{\vartheta\mathcal{X}^*_{\text{v}}\mu_0^2 e^4 \omega_1^2 m_1}{(4\pi)^2\frac{1}{2}\rho m_2}\frac{1}{r^2}(-\hat{r}) \qquad (11.10)$$

$\mathbf{F}_{m.\text{pol.1 on }2}(r_2)$ *is always directed toward particle 1, as indicated by its minus sign, and accordingly is always an attractive force.* We next regard charge 2 as the source and 1 as the test charge. Going through similar procedure as for mass 1 above we get the RFVP Lorentz force acted by mass 2 on mass 1:

$$\mathbf{F}_{m.\text{pol.2 on }1}(r_1) = -\mathbf{F}_{m.\text{pol.1 on }2}(r_2) = \frac{\pi}{4}\frac{\vartheta\mathcal{X}^*_{\text{v}}\mu_0^2 e^4 \omega_2^2 m_2}{(4\pi)^2\frac{1}{2}\rho m_1}\frac{1}{r^2}(\hat{r}) \qquad (11.11)$$

It is directed toward mass 2. Thus the common attractive, RFVP Lorentz force between masses m_1 and m_2 is

$$F_g = |\mathbf{F}_{m.\text{pol.1 on }2}(r_2)| = |\mathbf{F}_{m.\text{pol.2 on }1}(r_1)|$$

$$= \sqrt{|\mathbf{F}_{m.\text{pol.1 on }2}(r_2)||\mathbf{F}_{m.\text{pol.2 on }1}(r_1)|} = \frac{\pi}{4}\frac{\vartheta\mathcal{X}^*_{\text{v}}\mu_0^2 e^4}{(4\pi)^2\frac{1}{2}\rho}\sqrt{\frac{\omega_2^2 m_2 \omega_1^2 m_1}{m_1 m_2}}\frac{1}{r^2}$$

or

$$F_g = \frac{\pi}{4}\frac{\vartheta\mathcal{X}^*_{\text{v}}\mu_0^2 e^4}{(4\pi)^2\frac{1}{2}\rho}\omega_2\omega_1\frac{1}{r^2} \qquad (b)$$

Substituting into (b) with the identities $\hbar\omega_1 = m_1c^2$ and $\hbar\omega_2 = m_2c^2$ given by applying $(6.5)'$ to the two charges, we have

$$F_g = \frac{\pi}{4}\frac{\vartheta\mathcal{X}_{\rm v}^*\mu_0^2e^4c^4m_1m_2}{(4\pi)^2\frac{1}{2}\rho\hbar^2}\frac{1}{r^2}$$

or

$$F_g = G\frac{m_1m_2}{r^2} \tag{11.12}$$

where

$$G = \frac{\pi}{2}\frac{\vartheta\mathcal{X}_{\rm v}^*\mu_0^2e^4c^4}{(4\pi)^2\rho\hbar^2} \tag{11.13}$$

F_g of (11.12) is identifiable as Newton's gravitational force; accordingly G of (11.13) is the Newtonian constant of gravitation. It can be readily checked that G has the desired dimension, $[G] = \mathrm{m}^3\ \mathrm{kg}^{-1}\ \mathrm{s}^{-2}$.

11.2.2 Gravitational force between compound particles or bodies

We consider now two electrically neutral bodies of masses m_a and m_b, with their centers of mass located at r_a and r_b receptively; each body is composed of N_t (long-lived) basic material particles (i.e. electrons and protons) or the compounds of the two spices, $t = a, b$. When reduced to microscopic scale the two bodies then represent compound particles (like atoms or molecules). Until the justification in Sec. 11.3, we assume for now that the vacuum depolarization field and magnetic field due to a basic particle i of mass a, $\mathbf{E}_{\mathrm{pol}.ai}$ $\mathbf{B}_{q.ai}$ are capable to penetrate through any mass on its path, whilst the free-charge radiated field $\mathbf{E}_{ai}$ may not necessarily do so. Then the total attractive Lorentz force, or the gravitationa froce, between the two bodies is given by the sum of the RFVP Lorentz forces between all particles of a with all partciles of b, each given by (11.12); the constant G of (11.13) is the same for all the interaction pairs. That is

$$F_{g.ab} = \sum_{i,j=1,1}^{i,j=N_a,N_b} F^m_{\mathrm{pol}.ai\ \mathrm{on}\ bj} = G\sum_{i,j=1,1}^{i,j=N_a,N_b}\frac{m_{ai}m_{aj}}{\left[(\mathbf{r}_{bj}+\boldsymbol{\xi}_j)-(\mathbf{r}_{ai}+\boldsymbol{\xi}_i)\right]^2} \tag{11.14}$$

Where $\boldsymbol{\xi}_j$ and $\boldsymbol{\xi}_i$ are distances of basic particles j and i to their respective centers of mass of a and b. Supposing $\boldsymbol{\xi}_j << r, \boldsymbol{\xi}_i << r$, which is the standard assumption for the basic Newton's law of gravitation for two bodies, then (11.14) simplifies to

$$F_{g.ab} = \sum_{i,j=1,1}^{i,j=N_a,N_b} F^m_{\mathrm{pol}.ai\ \mathrm{on}\ bj} = G\frac{\sum_i^{N_a} m_{ai}\sum_j^{N_b} m_{bj}}{r^2} = G\frac{m_am_b}{r^2} \tag{11.14}$$

where

$$m_a = \sum_i^{N_a} m_{ai}; \qquad m_b = \sum_j^{N_b} m_b; \qquad r = |\mathbf{r}_b - \mathbf{r}_a|;$$

and G is given by (11.13).

11.3 Mechanism for a non-dissipative induced electric field $\mathbf{E}_{\text{pol}.ai}$ and magnetic field $\mathbf{B}_{q.ai}$

We identified in Chapter 3 that the electric field, $\mathbf{E}_q$, from a free charge in vacuum corresponds to the transverse stress caused by the mechanical displacement of the neutral vacuons. And this stress originates from short range repulsion of the n-vaculeon envelops of the neutral vacuons. Suppose we now place a sufficiently "heavy" object– a piece of condensed matter, as a screen, and supposing its screening effect is tight. Then the redistribution of vacuons and the resultant stress on one side of the screen will not be transmitted to the other side. Consequently, the $\mathbf{E}_q$ will be readily reflected by matter, or absorbed by it. This anticipates the radiation pressure due to $\mathbf{E}_q$ will be in principle a surface effect; most of the waves impinging on to a body will be principally reflected at the surface of it. This phenomenon is typically observed in experimental observation.

Some waves will penertaa distance into the material. But

11.4 Estimate the magnitude of vacuon mass

Rearrangeing (11.13) we get the expression for an effective quantity, ρ_{ef}, of the linear density ρ of the apparent vacuon chain through which the particle's constituent electromagnetic waves propagate:

$$\rho_{\text{ef}} = \frac{\rho}{\vartheta \mathcal{X}_{\text{v}}^*} = \frac{\pi}{2} \frac{\mu_0^2 e^4 c^4}{(4\pi)^2 \hbar^2 G} \tag{11.15}$$

Where the final expression contains only constant or fundamental constants of values:

$$\mu_0 = 4\pi \times 10^{-7} \ \ \text{NA}^{-2}; \qquad e = 1.602 \times 10^{-19} \ \ \text{C}; \qquad c = 3.0 \times 10^8 \ \ \text{m/s};$$
$$\hbar = 1.054 \times 10^{-34} \ \ \text{Js}; \qquad G = 6.67259 \times 10^{-11} \ \ \text{m}^3\text{kg}^{-1}\text{s}^{-2}$$

Substituting the above values into (11.15) we have

$$\rho_{\text{ef}} = 1.1305 \times 10^{23} \ \ \text{kg/m} \tag{11.15$'$}$$

With this in the left-hand side of (3.69) we get an effective quantity, $\rho_{\text{v}\cdot\text{ef}}$, of the volume density of vacuum ρ_{v}:

$$\rho_{\text{v}\cdot\text{ef}} = \frac{\rho_{\text{v}}}{\vartheta \mathcal{X}_{\text{v}}^*} = \frac{\rho_{\text{ef}}}{\pi \frac{16}{3} b^2} = 1.68 \times 10^{57} \ \ \text{kgm}^{-3} \tag{11.16}$$

where $b = 4 \cdot 10^{-18}$ m as estimated in (2.4). With (11.16) we further estimate the effective mass of a vacuon to be

$$\mathcal{M}_{\text{v}\cdot\text{ef}} = \rho_{\text{v}\cdot\text{ef}} b^3 = 1.349 \times 10^4 \ \ \text{kg} \tag{11.17}$$